Optimal concession contracts for oil exploitation

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Abstract

This paper studies the relationship between a government and private companies for the exploitation of an oilfield by means of concession-like contracts, i.e. concessions and Production Sharing Agreements. At this aim, we develop and solve a dynamic stochastic optimization problem in a real option framework. The model takes into account crucial as well as actual features of the real world, such as: the twofold goal of governments who must mediate between social interests and revenue maximization from concessions; the incentive for the private party to "over exploit" natural resources and uncertainty over future payoffs. The results obtained can help policy makers in pursuing the delicate task of setting the "right" terms of concession-like contracts, meaning that policy makers can have at least a benchmark to start interacting with private parties. This phase is particularly difficult for a number of reasons, such as the need to trade contrasting interests off, high risk of corruption and the fact that negotiations are made difficult by the high level of uncertainty due to incomplete or even faulty information.

Keywords: Oil; Concession contract; Production Sharing Agreement; Real options.

1 Introduction

Many governments sign contracts with foreign companies to develop and sell their natural resources; therefore, negotiating the right contract is crucial to a government's efforts to pursue its ultimate goal

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to raise funds for social and economic development. This article studies formally the relationship between a public body – henceforth, a government – and a variety of private companies who engage in contracts for the exploitation of natural resources. Due to its relevance and its peculiar features, as a natural resource we focus on the specific case of oil. From a methodological perspective, we follow a dynamic stochastic optimization approach in the framework of real options (RO) (see Dixit and Pindyck, 1994 for a detailed and rigorous treatment of this mathematical instrument). Along this route the model provides closed form formulae of some key variables of the contracts, such as the minimum bidding price and the optimal entry thresholds. The results are achieved by taking into account crucial as well as actual features of the real world, such as: the twofold goal of governments who must mediate between social interests and revenue maximization from concessions; the incentive for the private party to "over exploit" the natural resources and uncertainty over future payoffs. Uncertainty over future payoffs can be considered as a sort of powerful synthesis of many sources of uncertainty that oil exploration and exploitation invariably pertains. Notably, political, geological and engineering uncertainties are intrinsically related and reciprocally entangled such that all the sources of uncertainty eventually affect the expected payoff from the investment (see Cequeti and Ventura, 2015). For this reason, over the last years we are witnessing a growing body of multidisciplinary articles on the topic, for instance Dai et al (2020), Ampomah et al (2017), Iskandarani et al (2016) and the literature cited therein, just to cite a few. The results obtained from the model are quite useful both for governments and companies: indeed, setting the "right" terms of the concession contracts – or, at least, having a reliable benchmark to start with – is of crucial importance for a number of reasons. First, the government is expected to protect the public interest, e.g. ensuring that oil spills do not damage environment; at the same time, it is also expected to create more favourable economic conditions promoting investments and job opportunities. These goals are sometimes seemingly in conflict with each other – at least in the short run – and contractual terms should somehow trade this contrast off. Second, contracts in energy deals are characterized by huge investments costs and large profits, which attract opportunities for corruption. It is common practice to disclose so little information about negotiations and contract terms, giving rise to potential for abuse on both sides of the table. Actually, corruption in concession contracts is not exclusive of energy field, but it pertains any other sector of the economy where contracts involve a huge amount of public money. Indeed, Global Witness (2004) reports that corruption in concession contracts has been pursued in Angola, Congo Brazzaville, Kazakhstan, and elsewhere. Third, negotiations are made difficult by the high level of uncertainty due to incomplete or even faulty information.

The remainder of the paper is organized as follows. A short description of the most widespread concession contracts for the exploitation of oil and other natural resources is given in Section 2. This section helps identifying the critical aspects of concession contracts that we want to capture in the modelling exercise. A detailed literature review is given in Section 3. The model is introduced in Section 4 and its solution is contained in Section 5. Section 6 offers some conclusive remarks, with policies implications and lines for future research.

2 Salient features of concession contracts in brief

One of the first decisions a government must make to develop its natural resources, is to choose the type of contractual system basically between the following three options: a concession or license agreement, a Production Sharing Agreements (PSA), or a joint venture. Admittedly, the provision of license-concession agreements and PSAs resemble each other and – being by far the most widespread contracts – we will devote our efforts to study and modeling them.

In this respect, notice that the specific analysis of PSA is *per se* a worth and relevant research theme, and there are still many realities dealing with PSA as unique device for concession agreements in the context of fossil fuels. In this respect, some noticeable and recent examples can be presented. Weijermars and Zhai (2016) focus on the comparison of the contractual systems in Mexico and US for the specific case of hydrocarbon reservoirs. The authors discuss the realities of the considered countries – which are based on contracts between contractor and government – and present the discrepancies of PSA allocation strategies between the two parties for Mexico and US. Interestingly, the paper acknowledges the existence of other sources of agreements, but it explores only PSA – which is the one used for the shallow offshore exploitation of the natural resource. Mariano et al. (2018) cleary state that PSA is the most commonly used contractual system. The authors present the case of the concessions related to the exploitation of oil and natural gas in Brazil, and give a clear view on how PSA is in force in that country. The details on the regulation behind the statement of PSA are also provided. Alrishani (2020) is a very recent contribution dealing with a complete description of the PSA and discussing the particular case of the OiloGo Inc., a large energy company playing the role of contractor in several cases.

For given concession-like contracts, governments have three options: they can create State companies, as in Saudi Arabia, Mexico, Venezuela, Iran and Oman. They can allow private investors to develop the natural resources, as in the US, the UK, Russia and Canada. Or, they can use a combination of these two systems, as in Indonesia, Nigeria, Azerbaijan and Kazakhstan (Open Society Institute, 2005).

Concessions grant a company, or more than one company, the exclusive rights to explore, develop, sell and export oil in a specific area for a fixed period of time. Companies compete by offering bids for the license to such rights. The financial and other terms of the license are set forth in an agreement drafted by the host government, which should then be published and opened to a bidding process by competitive companies. The successful bidder pays the bidding price and the fees are kept by the host government, regardless of whether oil is found and commercial production takes place. If commercial production occurs, the host government also earns royalties based on gross revenue and/or a profit tax based on net income, both of which are based on the quantity of production and the price at which the production is sold.

An important feature of concession contracts and PSA is that the government may want to incorporate some contractual provisions aimed at protecting social interests – such as environmental and other standards – being it charged to protect and pursue the local interest. Agreements can be terminated, for example, for repeated environmental violations or if companies are no longer developing the field. Therefore, host governments require a guarantee in order to hold reliable contractual counterpart with the resources to cover potential liabilities.

From this brief description of the dynamics leading the parties to the signature of concession contracts, it clearly emerges the need to take into account the following features: (i) the governments must mediate between social interest, especially with regards to environmental damage, and revenue maximizing behaviour; (ii) concession holders have the incentive to over exploit the resource; (iii) governments should envisage contingent counter actions to cover potential liabilities. With respect to the existing literature, our model provides a new contribution because it puts these elements together. Therefore, the final results – achieved in terms of price and optimal entry thresholds – can be considered as a good starting point for the definition of contractual terms, making them more transparent and shrinking room for abuse on both sides of the contract.

3 Review of the literature

In spite of the huge number of papers on natural resource exploitation, RO hinge on three pillar assumptions that make such a theory as the natural candidate to model the optimal use of natural resources and – consistently – concession contracts. The three pillars are: (*i*) the irreversibility component of the investment considered; (*ii*) the uncertainty surrounding the expected returns from the investment; (*iii*) the term structure of the concession, i.e. the expiration date. For this reason – and for sake of room – in what follows we review only the contributions within the RO paradigm. Within this approach, we identify three streams: contributions dealing with the issue of natural resources in general – without special attention on oil; contributions dealing with the specific case of oil; contributions pertaining to concession contracts, for any type of resource exploitation. Obviously, some articles can straddle more streams. In this respect, the simplest case is represented by articles dealing with the evaluation of petroleum leases.

The application of RO to natural resources dates back to the seminal and almost simultaneous works of Pindyck (1984) and Brennan and Schwartz (1985). The former studied the effect of uncertainty in renewable resource prices on competitive equilibrium in a market with property rights. The latter applied the theory to evaluate a firm operating in the mining sector, in a monopolistic context, which can be easily regarded as a concession holder. Successively, Morck et al (1989) applied RO theory to evaluate a logging company holding a concession contract in Canada, whereas Schwartz and Trigeorgis (2001) presented a number of applications to natural resources. These contributions spawned a copious flow of research.

In the second strand of the literature – i.e., the specific case of oil – one of the very first contributions is attributable to Paddock et al. (1988), who applied the option valuation theory to value leases of offshore petroleum. Smith and McCardle (1996, 1999) developed a model of an oil property to study the optimal suspension, the decision-making process for exploration and development, and the optimal time to invest. Laughton (1998) found that oil price uncertainty delays all option exercises, whereas exploration and delineation occur sooner with reserve size uncertainty. Tang et al. (2017) discussed RO, decision-tree and Monte Carlo simulation in oil applications. Chorn and Croft (2000) studied the value of reservoir information.

The contributions belonging to the third stream (Pindyck, 1984; Brennan and Schwartz, 1985; Morck et al., 1989; Saito et al., 2001; D'Alpaos et al, 2006, Monjas-Barroso and Balibrea-Iniesta, 2013) share the idea that the value of the concession is given by the expected net present value of future payoffs, plus the option to delay the investment, and aim at evaluating the contract and/or the optimal time to invest and/or managerial flexibility. Fan and Zhu (2014) took the stance of an oil company who wants to determine the value of oil-resources before deciding whether to apply for a concession. Others, focused on very specific aspects of concession contracts, for instance on the value of the penalty fee, i.e. a fee charged to concessionaire should it not provide the goods/services agreed by the scheduled date (see D'Alpaos and Moretto, 2013), should it terminate participation in the contract early (see Huang and Pi, 2014), or should it put into place moral hazard behavior (Wang and Pallis, 2014). Last, but not least, few contributions focused on concessions before being awarded – namely, few devoted attention to determine the crucial contractual terms before reaching an agreement between a government and concession holders. In a recent contribution, Jin et al. (2019) used RO theory and Monte Carlo simulations to calculate jointly the value of the concession period and the minimun revenue guarantee to satisfy both public and private parties' interests. Scandizzo and Ventura (2010) analyzed the interactions between a government and a possible unique concessionaire. Caselli et al. (2009) analyzed the specific issue of valuing the buyout clause – i.e., the possibility for the public partner to redeem a concession from the private partner before the end of the concession period. This is a question that has to be solved before signing the concession agreement, as this clause has to be included in the concession agreement.

Our contribution is nested exactly within this strand of the literature, attempting to move a little but significant step ahead. Indeed, we model the alternative choices a government faces when deciding to award concession contracts – i.e., it can assign the project to a State owned company, the so called National Oil Company (NOC), such as for Kuwait Oil Company in Kuwait and Saudi Aramco in Saudi Arabia; it can assign a certain number of concession contracts to foreign oil companies with or without the concurrence of a NOC. In turn, both alternatives must compared to a third choice which consists in doing nothing, i.e. not assigning any licence contract and preserving the natural resource as it is. Each of these possible choices has its own costs and benefits; a straightforward comparison provides decision makers with benchmarks to make sound decisions complying with public and private interest.

4 The model

We take a probability space with filtration $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ which contains all the random quantities introduced hereafter.

Outline of the problem

We consider n private companies that can be potentially assigned a license each, for the exploitation of an oilfield.

The cash flow of the project is supposed to follow a stochastic process denoted by $X = \{X(t)\}_{t \geq 0}$, and it is assumed to evolve according to a Geometric Brownian Motion:

$$dX(t) = \alpha X(t)dt + \sigma X(t)dW(t) \qquad \forall\, t > 0; \quad X(0) = X, \tag{1}$$

where $\{W(t)\}_{t \geq 0}$ is a Brownian Motion, $\alpha > 0$ and $\sigma > 0$ are the drift and the instantaneous volatility rate of the cash flow, respectively, and $X > 0$ is its initial value at time $t = 0$, representing the current value of the cash flow.

The license holders are entitled to share the net cash flow generated by the exploitation of the resource. Very often, in the context of oil there is a $(n + 1)$-th potential license holder which is of public nature, a NOC, which, under certain circumstances, could be the unique firm entitled to the exploitation of the oilfield, i.e. the only license holder.

We take the stance of a government that is choosing between one of the following three mutually exclusive possibilities: (i) preserving the resource as it is, i.e. not assigning any license; (ii) assigning exclusive rights to the NOC; (iii) assigning licenses to the $n + 1$ companies, the n private plus the NOC.

4.1 Details of the problem

The payoffs accruing to the government from the three possible choices are denoted as: $B(0), B(1)$ and $B(n + 1)$ as in the cases (i), (ii) and (iii) just mentioned, respectively. Let us start from $B(0)$.

Choosing not to develop the resource now does not prevent the government from doing it in the future. Put another way, keeping the resource in the *status quo* now, makes the government acquire an option to develop it in the future. In order to keep things as general as possible, and in line with Scandizzo and Ventura (2010) we also assume that, in the absence of development, the resource yields a steady flow of economic benefits (of the public amenity type) net of maintenance cost, and that such a flow would be lost in the event of development. While different hypotheses would be possible in this regard, this particular assumption seems well-suited to the case of many contracts that involve natural resources, such as a park, a wildlife area etc. In most of these cases, the concession provides for a period of privatized management of the public space involved, with development consisting of an infrastructure that permanently reduces the flow of amenities in the interested area. This hypothesis is also interesting because it explores an important dimension of the concession contract, that is, a temporary privatization combined with a development project involving some permanent loss of a public amenity. It follows that, the value of doing nothing now, $B(0)$, is given by the value the resource yields in the current state plus the option to develop it in the future. This simple claim is modelled in equation (2)

$$B(0) = \int_0^\infty y e^{-\rho s} ds + F_0(X_0^{(e)}) = \frac{y}{\rho} + F_0(X_0^{(e)}), \tag{2}$$

in the right-hand-side, RHS, of (2) y represents the deterministic instantaneous flow of public amenities discounted at rate ρ, while the option to develop the resource in the future is a function of a threshold $X_0^{(e)}$ and it is denoted by $F_0(X_0^{(e)})$. Specifically,

$$X_0^{(e)} = \operatorname{argsup}_{X>0} \{F_0(X)\}. \tag{3}$$

An explicit expression for F_0 will be presented below, on the basis of dynamic programming arguments and, accordingly, the value of $X_0^{(e)}$, – which makes optimal to exercise the option – will be endogeneously determined. $X_0^{(e)}$ is commonly referred to as "optimal entry threshold" or, simply, "entry threshold" for the case of no license assigned. Details about dynamic programming and how to obtain the value of $X_0^{(e)}$ and the shape of F_0 are reported in Section 5. In a very similar manner we can set the value of $B(1)$ equal to the expected discounted value of the cash flow accruing to the NOC, which ultimately coincides with the government, from the developed resource minus the sunk cost of the investment. Formally,

$$B(1) = \mathbb{E}\left[\int_0^\infty X(s) e^{-\rho s} ds\right] - I, \tag{4}$$

where I is the sunk cost borne by the NOC.

Eventually, when considering $B(n+1)$ one must think that the government earns a fraction of the cash flow generated by the investment and accruing to the NOC over the licenses duration, plus the entire cash flow once expired the n licenses assigned to the private companies, plus the price paid by the companies to gain the licenses, minus the investment cost borne by the NOC. In addition to these quantities one must also take into account the fact that the government retains the task and duty of safeguarding local interests and may want to incorporate some contractual provisions aimed at protecting it, as extensively explained in Section 1. This occurrence is captured in the model by adding an extra term. In this case, the formalizazion of the expression for $B(n+1)$ can help making things clear:

$$B(n+1) = \mathbb{E}\left[\gamma \int_0^T X(s)e^{-\rho s}ds\right] + \mathbb{E}\left[\int_T^\infty X(s)e^{-\rho s}ds\right] + F_1(X_1^{(e)}) + P - I, \tag{5}$$

where $\gamma > 0$ is the share of the expected cash flow accruing to the NOC, and the first expected value term is the cash flow accruing to the NOC. Such a revenue is in force only over the concession period $[0, T]$ while, after T the resource goes back to the government which then takes over the entire amount, captured by the second term in the square brackets. P represents the revenues from the prices paid by the n private firms, I the sunk cost of investments borne by the NOC and the term F_1 captures the option to undertake some action against the private companies in order to safeguard local interests, with a reference to the entry threshold $X_1^{(e)}$ which will be optimally determined (see the details in Section 5). Notice that F_1 takes the form of an option because the government keeps the right not the obligation to intervene. Without loss of generality, this term can be regarded as capturing the fact that the host government requires a guarantee from the companies so that it holds a reliable contractual counterpart with the resource to cover potential liabilities. Indeed, for that reason the value of F_1 is contingent upon the cash flow, and in particular it is an increasing function of it. Actions jeopardizing local and social interests, on the part of the private party, are obviously undertaken in order to increase the net gain from her investment. It follows that violations of social interests become more likely as the underlying asset increases. It follows that the government's counteraction becomes more likely as X_t increases, and this requires F_1 to be an increasing function of X_t. Also in this case, as in (3), we have

$$X_1^{(e)} = \text{argsup}_{X>0} \left\{ F_1(X) \right\}. \tag{6}$$

Mutatis mutandis, the same arguments developed for F_0 apply to F_1. An explicit formulation for F_1 and the optimal entry threshold $X_1^{(e)}$ in (6) will be derived in strict analogy to F_0.

Fubini's Theorem and Eq. (1) allow to rewrite $B(1)$ and $B(n+1)$ as

$$B(1) = \int_0^\infty \mathbb{E}\left[X(s)\right] e^{-\rho s} ds - I = \int_0^\infty X e^{-\delta s} ds - I = \frac{X}{\delta} - I \tag{7}$$

and

$$B(n{+}1) = \gamma \int_0^T \mathbb{E}\left[X(s)\right] e^{-\rho s} ds + \int_T^\infty \mathbb{E}\left[X(s)\right] e^{-\rho s} ds + F_1(X_1^{(e)}) + P - I =$$

$$= \gamma \int_0^T X e^{-\delta s} ds + \int_T^\infty X e^{-\delta s} ds + F_1(X_1^{(e)}) + P - I =$$

$$= \frac{X}{\delta} \left[\gamma(1 - e^{-\delta T}) + e^{-\delta T}\right] + F_1(X_1^{(e)}) + P - I. \tag{8}$$

where the discount rate $\delta = \rho - \alpha$ is the so called implicit *convenience yield*. The oil convenience yield can be interpreted as the interest rate, denominated in barrels of oil, for borrowing a single barrel of oil, and it measures the value of storing crude oil over the borrowing period. The greater the value of δ, the greater the discount rate in (7) and (8) and the lower the value of the options, because the government is impatient to exercise them, i.e. it does not want to wait long to invest, because investing it can gain a high implicit dividend.[1]

5 Solving the problem

We want to define the conditions under which the government makes the optimal the choice of developing an oilfield granting $n+1$ concession contracts, because this is the most widespread situation around

[1] For most commodities, convenience yield varies inversely with the total amount of storage. There is a broad literature about convenience yield; for pioneering empirical studies about its role in price formation see Pindyck (1993, 1994), for recent works on its predictive power see Gaspodinov and Ng (2013), Fernandez (2020) and the literature therein.

the world. Very simply, the government will prefer assigning one license than doing nothing whenever $B(1)$ is at least as worth as B(0). Similarly, the government will prefer assigning $n+1$ licenses whenever $B(n+1)$ is at least as worth as $B(1)$. Thus, in what follows we will first analyse the condition under which $B(1) \geq B(0)$, then $B(n+1) \geq B(1)$ and eventually we will discuss the conditions under which the double inequality $B(n+1) \geq B(1) \geq B(0)$ holds.

5.1 One license *vs* no license

To figure out the condition under which $B(1) \geq B(0)$ we adapt from Scandizzo and Ventura (2010). In this specific set up, we can compare the RHS of (7) and (2). Notably, it is possible to write the condition of indifference between the two alternatives as:

$$\frac{X}{\delta} - I = \frac{y}{\rho} + F_0(X). \tag{9}$$

An explicit expression for the values of the options F_0 in (9) can be derived by following the approach by Dixit and Pindyck (1994, pp. 122-123). Along this line it is also possible to obtain the optimal threshold $X_0^{(e)}$. In particular, the no-arbitrage condition reads as follows:

$$\mathbb{E}\left[dF_0(X)\right] = \rho F_0(X)dt. \tag{10}$$

By applying Ito's lemma, taking the expected value and by (1) we can rewrite (10) as follows:

$$\alpha X F_0'(X) + \frac{\sigma^2 X^2}{2} F_0''(X) = \rho F_0(X). \tag{11}$$

Which is a homogeneous second order ordinary differential equation, the general solution of which is of the type

$$F_0(X) = A_1 X^{\beta_1} + A_2 X^{\beta_2}, \tag{12}$$

where

$$\beta_1 = \frac{-\alpha + \frac{\sigma^2}{2} + \sqrt{(\alpha - \frac{\sigma^2}{2})^2 - 2\rho\sigma^2}}{\sigma^2} \tag{13}$$

and

$$\beta_2 = \frac{-\alpha + \frac{\sigma^2}{2} - \sqrt{(\alpha - \frac{\sigma^2}{2})^2 - 2\rho\sigma^2}}{\sigma^2} \tag{14}$$

are respectively the positive and greater than unity, and the negative root of the associated characteristic equation

$$\frac{\sigma^2}{2}\beta(\beta - 1) + \alpha\beta - \rho = 0 \tag{15}$$

and A_1 and A_2 are two constants that can be determined from the boundary conditions. First, notice that when the underlying is zero the option to enter an investment must be worthless, i.e. $F_0(0) = 0$, thus $A_2 = 0$ in (12), which boils down to

$$F_0(X) = A_1 X^{\beta_1}. \tag{16}$$

The remaining constant and the threshold value, A_1 and $X_0^{(e)}$, can be determined by means of the boundary conditions. In dynamic programming the boundary conditions are referred to as the value matching and its derivative, the so called smooth pasting. Generally, the value matching matches the value of the unknown function F_0 to that of the termination payoff function, namely the expected value of the investment. In our specific case the value matching is nothing but Eq. (9), which matches the values of the two alternative strategies.

Therefore, the two boundary conditions can be written as:

$$\begin{cases} \frac{X}{\delta} - I = \frac{y}{\rho} + A_1 X^{\beta_1} \\ \frac{1}{\delta} = A_1 \beta_1 X^{\beta_1 - 1} \end{cases} \tag{17}$$

being the first relation in (17) the value matching condition and the second the smooth pasting one and with β_1 as in (13). Multiplying the smooth pasting by X and dividing by β_1 we obtain

$$\frac{X}{\delta\beta_1} = A_1 X^{\beta_1} \tag{18}$$

The RHS of this expression can be substituted back into the left-hand-side, LHS, of the value matching and then solved for $X = X_0^{(e)}$ which is the maximizing value of the option term:

$$\frac{X}{\delta} - I = \frac{y}{\rho} + \frac{X}{\delta\beta_1} \tag{19}$$

$$X_0^{(e)} = \frac{\delta\beta_1}{(\beta_1 - 1)} \left(\frac{y}{\rho} + I \right) \tag{20}$$

In turn, (20) can be used to obtain the value of the constant A_1

$$A_1 = \left(\frac{\frac{y}{\rho} + I}{\beta_1 - 1} \right)^{1-\beta_1} [\delta\beta_1]^{-\beta_1} \tag{21}$$

$X_0^{(e)}$ in (20) is the minimum value taken by the cash flow dynamics that makes $B(1)$ preferable to $B(0)$. Formula (20) has a clear cut interpretation. The term $\beta_1/(\beta_1 - 1)$ is greater than 1 and accounts for risk. Indeed, in order to make the government willing to enter the investment abandoning the *status quo*, the discounted value of the expected cash flow at the time of entry, i.e $X_0^{(e)}$ must be greater than the costs that the government bears, and the magnitude of this difference is greater the greater the uncertainty associated with the investment, $\partial X_0^{(e)}/\partial\sigma > 0$ given that β_1 is a function of σ and – according to (13) – one has $\partial\beta_1/\partial\sigma < 0$. Thus, in this case the greater the uncertainty, the greater the incentive for the government to delay the investment.

5.2 $n + 1$ licenses *vs.* one license

We discuss here when $B(n + 1) \geq B(1)$.
From the RHS of (8) and (7) one can write the condition of indifference between the two alternatives as:

$$\frac{X}{\delta} \left[\gamma(1 - e^{-\delta T}) + e^{-\delta T} \right] + F_1(X) + P - I = \frac{X}{\delta} - I \tag{22}$$

Simplifying and rearranging terms:

$$F_1(X) = \frac{X}{\delta} \left[(1 - \gamma)(1 - e^{-\delta T}) \right] - P \tag{23}$$

The LHS of (23) contains the option F_1 the value of which can again be determined again by dynamic programming. It follows that an explicit formulation of $F_1(X)$ is given by $B_1X^{\beta_1}$, where B_1 is a constant to be determined from the boundary conditions and β_1 is as in Section 5.1. The value of the constant B_1 and the optimal threshold $X = X_1^{(e)}$ – which represents the specific value of the process which makes the government exercise the option optimally – can be determined by considering the value matching and its derivative. Indeed, (23) can be taken as a value matching because it matches the value of the unknown function F_1 to that of the termination payoff function, namely the expected value of the investment from granting $n + 1$ licenses instead of only one license to the NOC. Therefore, the two conditions can be written as:

$$\begin{cases} B_1X^{\beta_1} = \frac{X}{\delta}\left[(1-\gamma)(1-e^{-\delta T})\right] - P \\ B_1\beta_1X^{\beta_1-1} = \frac{1}{\delta}\left[(1-\gamma)(1-e^{-\delta T})\right] \end{cases} \tag{24}$$

being the first relation in (24) the value matching condition and the second the smooth pasting.
Multiplying the smooth pasting by X and dividing by β_1 we obtain

$$B_1X^{\beta_1} = \frac{X}{\delta\beta_1}\left[(1-\gamma)(1-e^{-\delta T})\right] \tag{25}$$

The RHS of this expression can be substituted back into the LHS of the value matching and then solved for X, so that we obtain $X = X_1^{(e)}$ as follows[2]

$$X_1^{(e)} = \frac{\delta\beta_1 P}{(1-\gamma)(1-e^{-\delta T})(\beta_1 - 1)}. \tag{26}$$

The threshold $X_1^{(e)}$ is the specific value of the underlying stochastic process X above which the government exercises the expropriation option. It can be seen as an exit threshold, and signals that local interests must be safeguarded by taking actions against the private parties. It is greater the greater the price of the licenses discounted and adjusted for risk, $P\left(\frac{\delta\beta_1}{\beta_1-1}\right)$, and it is smaller the greater the share of the cash flow accruing to the n private firms, $(1-\gamma)$, and the longer the concession length, T. This sensitivity analysis reveals that the government is less eager to expropriate the private party as long as the latter incurs higher

[2]We do not figure out B_1 because it is not interesting from the point of view of the economic problem we are analysing, but it is can be obtained as we have already done for A_1.

costs and or lower benefits. Indeed, a higher price on the one side, and a lower share $(1-\gamma)$ or T on the other side, represent higher costs and lower benefits to the private party, respectively. These less favourable conditions of the deal decrease the possibility of over exploitation, because it is costly, and make the government willing to wait longer before expropriating, i.e. higher $X_1^{(e)}$. Put another way, the less favourable the conditions to the private party are, the less likely the expropriation bell will ring.

5.3 $n+1$ licenses preferred to one licence, in turn preferred to no license

We now explore the condition under which the double inequality $B(n+1) \geq B(1) \geq B(0)$ holds. Actually, this condition is rather simple to obtain because it can be derived from imposing the exit threshold to be not lower than the entry threshold, i.e. $X_1^{(e)} \geq X_0^{(e)}$. Thus, by comparing (26) and (20) we have that such condition is equivalent to:

$$P \geq P_{n+1} = \left(\frac{y}{\rho} + I\right)(1 - e^{-\delta T})(1-\gamma) \tag{27}$$

P_{n+1} is a reservation price, in that it is the government's price that makes it willing to develop the oilfield assigning $n+1$ licenses by assuming that the optimal options thresholds are taken into account. The term in parenthesis $\left(\frac{y}{\rho} + I\right)$ is the cost of carrying out the investment, namely the cost of developing the resource which, in turn, is made up of two components. The direct investment sunk cost, I, and the indirect opportunity cost, y/ρ, captured by the discounted value of the foregone flow of amenities. Equation (27) is a very important one because it makes clear that the condition which makes the government willing to forgo the public flow of amenities and the exclusive right to the NOC, by issuing $n+1$ licenses, can be stated in terms of the total revenue from the n private companies. This revenue, P_{n+1}, is greater the greater the cost of developing the resource, $\left(\frac{y}{\rho} + I\right)$, the share of cash flow granted to the private companies, $(1-\gamma)$, and the longer the concession period, T. At first sight it may seem puzzling that uncertainty does not play a role in this expression. Actually, this is due to the fact that P_{n+1} has been obtained under the circumstance that the options to abandon the *status quo* are in the money, as saying that the government has already accounted for uncertainty in its calculations.

6 Policy implications and further research

Many times concession agreements are renegotiated because infeasible conditions were set at the beginning – sometimes as a consequence of collusion between the parties. To a certain extent, we can claim that the results of the paper help to shed light on this point, proposing a transparent and applicable approach to determine the crucial conditions which make the deal not only feasible, but also optimal, from the government's point of view. To this regard, we highlight that a government's agency can directly apply the formulae coming from the model in writing the terms of the contract. Therefore, the model can be either a useful ex-ante tool, used by the policy maker when writing the terms of the contract – thus, having at least a transparent benchmark to start bargaining with private parties – , or as an ex-post tool. In this last case, should the final price the parties have agreed on be far below the reservation price in (27), an external observer would easily detect possible distortions, such as collusion between the parties. As a possible extension to our results, it would be interesting modelling the private parties' optimal decisions and then put together the two sides of the deal. In a context of game between the parties, the equilibrium price will be higher than the one in (27) and lower than the private party's reservation price, where the difference between the equilibrium price and (27) will be lower the higher the private party's bargaining power. This extension, would give the opportunity to formulate sound proposals for renegotiations, and re-analyse the results of suspected bargaining where needed.
Moreover, even if PSA is the most common type of concession contract (Mariano et al., 2018), we acknowledge the existence of other types of contractual systems different from PSA. It would be interesting to explore the framework presented here by adding them in the set of available opportunities. However, the addition of all the existing contractual systems in our theoretical setting would require a modeling approach

different from that presented here, hence deserving the writing of new papers. We leave this challenging topic for future research.

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