

# Shared Hybrid ARQ with Incremental Redundancy (SHARQ-IR) in Overloaded MIMO Systems to support Energy-Efficient Transmissions

Muhammad Kashif<sup>1</sup>, Muddesar Iqbal<sup>2</sup>, Zakir Ullah<sup>1</sup>, Tasos Dagiuklas<sup>2</sup>, Sohail Sarwar<sup>3</sup>, Zia Ul-Qayyum<sup>3</sup>, and Muhammad Safyan<sup>3</sup>

*Center for Advanced Studies in Engineering (CASE), Department of Electrical and Computer Engg, Pakistan<sup>1</sup>*

*School of Engineering, London South Bank University, UK SE1 0AA<sup>2</sup>*

*Allama Iqbal Open University, Islamabad, Pakistan<sup>3</sup>*

Corresponding author: Muddesar Iqbal (e-mail: m.iqbal@lsbu.ac.uk).

**ABSTRACT** Multiple Input and Multiple Output (MIMO) is a technology through which data is transmitted over the channel through multiple antennas. However, during its deployment and implementation, some pragmatic issues arise such as interference, multipath fading and noise leading to potential packet losses and consume substantial energy. In order address such issues, Hybrid ARQ transmissions provide effective means for error correction, especially in a noisy wireless channel. More often few bits in packets are found to be in error and it is unnecessary to use the entire MIMO channel for retransmission to correct the remaining errors. So a novel approach has been proposed in this paper i.e. Shared Hybrid ARQ (SHARQ - IR) using piggyback technique in overloaded MIMO systems where the transmitting antennas ( $N_t$ ) are more than the receiving antennas ( $N_r$ ) and used the concept of simple retransmission method to transform an overloaded MIMO into the critically loaded system ( $N_t=N_r$ ) or under loaded MIMO systems ( $N_t<N_r$ ). Simulation results outperform the contemporary approaches through reduced BER and a 20% throughput gain is observed during the simulation analyses which will ultimately support energy-efficient transmissions to encourage for Green IoT Applications.

**INDEX TERMS** Multiuser detection; MIMO; SHARQ; BER; Incremental Redundancy, Internet of Things (IoT)

## I. INTRODUCTION

Enabling Ultra-Low Latency applications is the need of time such as remote surgery, autonomous car and smart applications within the domain of Internet of Things (IoT). Due to the limited energy storage and resources of IoT devices, such applications demand high data rates, excessive bandwidth and near real-time connections and most importantly energy-efficient transmission. To fulfill their requirements, Fifth generation (5G) technologies have been launched. The road to 5G runs through 4G wireless infrastructure and improvement to 4G technologies such as massive multiple-input and multiple-output (MIMO) [1] and beam-forming is a promising technology that will achieve high data rate as required in 5G. MIMO system because of its substantial capacity and diversity gain is a key technology practically implemented in wireless standards such as IEEE 802.11n WLAN [2], IEEE 802.16

WiMAX [3] and LTE [4]. In recent years, researchers have addressed several MIMO challenges associated with these standard implementations [5-8]. However, there are some issues related to the MIMO system like performance and complexity, especially in overloaded scenario i.e. ( $N_t>N_r$ ). Generally, two techniques are used in MIMO detection: Linear detection and nonlinear detection. Zero forcing (ZF) [5] and MMSE [6] comes under the category of linear detection techniques. On the other hand, nonlinear MIMO detector relies on Maximum likelihood detection ML [7]. Zero force and MMSE performance degrades much in an overloaded MIMO system while ML is an optimal detection scheme that works well. However, during its implementation, increasing the users results in more computational power for data processing [8] [9]. In order to cater such issues, especially in overloaded conditions [10-12], low complex MUD (multi-users detection) techniques

have been proposed. Unfortunately considering the large size MIMO system deployed with hundreds of antennas, complexity increases exponentially. Hence, they are not viable for deployment in real time scenarios.

In order to increase the reliability of communication and reduce the computational complexity, various hybrid automatic repeat request (HARQ) schemes have been used in numerous wireless standards such as LTE. Different versions of HARQ schemes are available such as HARQ chase combining (HARQ CC) and HARQ incremental redundancy (HARQ IR). With HARQ CC [13], the same versions of the packets are transmitted during the retransmission and at the receiving end; the receiver then combines all the packets. In HARQ IR scheme [14], for each retransmission, the transmitter sends only redundant bits in incremental fashion for successful decoding. This increases reliability as incremental bits are transmitted only on a requirement basis. Moreover, most of the time few bits in a packet are found to be erroneous, so simply retransmitting the redundant bits results in bandwidth efficiency. Further to improve the bandwidth efficiency of MIMO channel, Space ARQ transmission (SHARQ) is used. Here, failed data packets are combined with new data frames during the retransmission using MIMO diversity. Similarly, Zahid et al. [15] have considered a MIMO system where  $N_t > N_r$  (over loaded) and have used two different schemes to convert this system to  $N_t = N_r$  (critically loaded) or to  $N_t < N_r$  (under loaded MIMO systems) by combining all retransmitted packets using HARQ CC [13].

Zahid et al. [15] has used the concept of HARQ CC on received stacked vector and considered it as a virtual receiver antenna [26]. We proposed a different approach by implementing SHARQ IR in an overloaded MIMO system and extended this concept to fully utilize the channel resources. SHARQ IR transmission mechanism can be used where the failed data packet is combined with the new packet and shared the same MIMO diversity channel over the physical layer. This gives an added advantage of achieving better BER and throughput as compared to the already implemented scheme using HARQ CC (as underlined by the simulation). Secondly, the proposed approach has an added advantage of improved performing in poor channel condition, where in addition to redundant bits' retransmission, new data packets are transmitted as well; that makes it a strong candidate for LTE and 5G communication standards.

In this paper, SHARQ IR is employed in the Overloaded MIMO system using a simple detection technique. This paper is the extension of our previous work where a simple scheme has been used for retransmission [27]. In that scheme, correctly decodable users remain idle during the next retransmission. However, to further improve the wireless channel utility, enhanced BER and throughput, SHARQ IR has been implemented by retransmitting the

redundant bits along with transmission of new bits for the correctly decodable users. We compare the simulation results of conventional HARQ with SHARQ IR for each number of retransmission. It has been found that SHARQ IR enhances the throughput by 20% and reduces BER by  $y\%$ .

The manuscript is organized as follows: In Section-II, the allied studies have been reviewed i.e. efforts carried out in the domain of MIMO technologies and formulated the system model in Section III. Section-IV elaborates the proposed approach and Section-V shows the simulation results by considering various parameters for BER and throughput. Section-VI concludes the work while highlighting potential research directions.

## II. Related Study

The first proposed algorithms for the detection of multi-users in MIMO systems have been the Diagonal Bell laboratories layered space-time (D-BLAST) and VBLAST [17-19], but D-BLAST algorithm is more intricate as compared to V-BLAST. V-BLAST provides optimum performance for multiple users but during detection, it leads to performance degradation by not considering the channel estimation error. Secondly, V-BLAST algorithm [10], due to its singularity nature, fails in an overloaded MIMO system and results in more errors during the detection process. Numerous optimum and suboptimum techniques have been also proposed for MIMO detection. The optimal techniques like ZF [20] and MMSE [20], due to their low complexity and simplicity for implementation, make them a strong candidate for MIMO detection but their performance drastically degrades in overloaded systems.

Hybrid ARQ transmission schemes are proposed in MIMO systems for enhancement in throughput [21]. This approach uses the forward-error correction (FEC) technique [22] for error detection and correction. HARQ IR protocol is more active for error correction compared to simple HARQ, and chase combining HARQ CC [23]. In [24], a linear precoder has been considered and joint HARQ detection has been carried out by combining all the stacked received vectors. However, only critically loaded conditions have been used in [24].

Although MIMO systems offer high spectral efficiency and diversity gain, they are more immune to channel noise and interference. While encountering channel distortion and fading, MIMO transmission often leads to packet loss. Moreover, most of the techniques focus on the retransmission keeping in view to fully utilizing the MIMO channel. However, practically only a few bits may be in error in the whole packet. Therefore, it is not efficient to reuse the entire MIMO channel for the same packet retransmission. It is therefore of great motivation to investigate how to fully utilize channel efficiency by employing SHARQ IR in the Overloaded MIMO system.

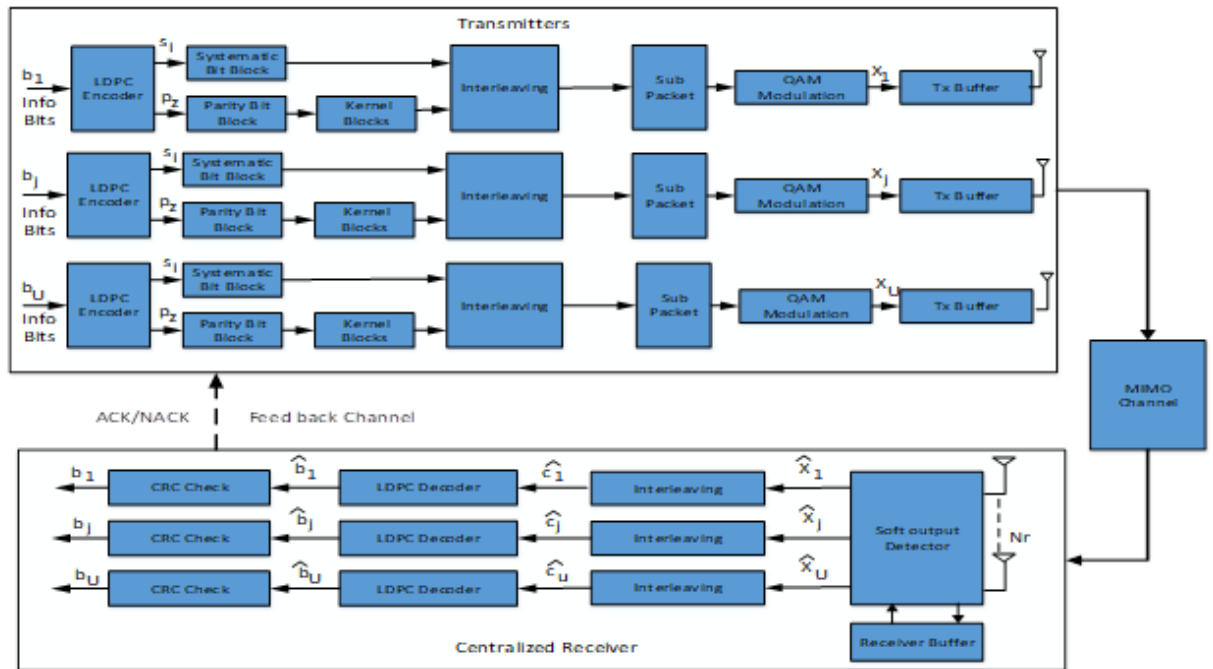


Figure 1: SHARQ-IR MIMO Architecture

### III. System Model

We considered ‘U’ users for an overloaded MIMO communication system, ‘Nt’ as transmit antennas and ‘Nr’ receive antennas. The correlation between transmitted signal and receive signal vector for ‘N’ transmission is given by:

$$y_i = H_i + x_i + n_i \quad i=1, 2, \dots, N \quad (1)$$

Where  $y_i$ ,  $H_i$  and  $n_i$  denotes the  $N_r \times 1$  received signal vector,  $N_r \times N_t$  is the channel matrix and  $N_r \times 1$  is the AWGN noise at time  $I$  respectively. The wireless channel unveils quasi-static, frequency-flat Rayleigh fading. The architecture of SHARQ-IR employed in this paper is shown in Figure 1.

The system model uses some information ‘K’ bits, which comprises of the data bits and CRC bits for error detection. Low-Density Parity-Check (LDPC) encodes the data bits at the rate of  $R_c=K/N$ . The rate at which the transmitted bits are encoded is referred to as mother code and generate a code packet. This is further classified into systematic S and parity bit P blocks. There are classified kernel blocks in the systematic and parity blocks, which have changed priorities to each other. The kernel blocks are classified in relation to the order in which the data bits are transmitted. There is a block-wise generation for systematic bits and parity bits. Encoded packet C is described as follow:

$$C = \{S; P\} = \{s_0, s_1, s_2, \dots, s_k; p_0, p_1, p_2, \dots\} \quad (2)$$

Where  $s_0, s_1, s_2, \dots, s_k$  are systematic bits of  $k^{th}$  order and  $p_0, p_1, p_2, \dots, p_z$  are parity bits of  $z^{th}$  order

Parity bit blocks are included in k-th kernel block set  $\Psi_k$ , ( $k=1, 2, \dots, 5$ ) are given as follows

1st kernel blocks: =  $\Psi_1 = \{P B1\}$ ; P B1 is the parity bits in Block 1

2nd kernel blocks: =  $\Psi_2 = \{P B2\}$ ; P B2 is the parity bits in Block 2

3rd kernel blocks: =  $\Psi_3 = \{P B3\}$ ; P B3 is the parity bits in Block 3

4th kernel blocks: =  $\Psi_4 = \{P B4\}$ ; P B4 is the parity bits in Block 4

5th kernel blocks: =  $\Psi_5 = \{P B5\}$ ; P B5 is the parity bits in Block 5

There is a shuffling of parity bits in each block. If a packet is found in error by the receiver during the transmission, a negative ACK will be sent to the transmitter requesting for the 1<sup>st</sup> kernel block. If the packet is still in error, the second negative ACK will be sent to the sender asking for sending of 2<sup>nd</sup> kernel block and this procedure will endure till the retransmission of last 5<sup>th</sup> kernel block. After the last kernel block i-e 5<sup>th</sup> kernel block, the packet is now considered as a dropped packet.

The shuffled coded bits and parity bits are then forwarded to the inter-leaver to generate a sub-packet. Here both the shuffled coded bits and priority bits are concatenated to form a Coded bit. The coded bits are then modulated by QAM modulation. The wireless channel after adding noise or

interference will forward it to the receiver. The receiver employs two different schemes for detection purposes. One is JML and the second one is Suboptimal MMSE detection

Joint Maximum likelihood detection is a technique used in MIMO system to minimize the probability of error at the receiver end and is given by:

$$L(c_p^u) = \log \left( \frac{P[c_p^u=1|y,H]}{P[c_p^u=0|y,H]} \right), \quad (3)$$

where  $u= 1, 2, 3, \dots, U$

$$= \log \left( \frac{\sum_{x \in X_p^1} \exp\left(-\frac{1}{\sigma_v^2} \|y - Hx\|^2\right)}{\sum_{x \in X_p^0} \exp\left(-\frac{1}{\sigma_v^2} \|y - Hx\|^2\right)} \right) \quad (4)$$

Applying log approximation [25] to above equation results in

$$L(c_p^u) = \frac{1}{\sigma_v^2} \left( \min_{x \in X_p^0} \|y - Hx\|^2 - \min_{x \in X_p^1} \|y - Hx\|^2 \right) \quad (5)$$

However, equation (5) is difficult to implement due to its complexity

The MMSE linear detector has low intricacy and shows acceptable performance. The received signal is given by

$$\hat{X}_{\text{MMSE}} = (H^H H + \sigma_v^2 I)^{-1} H^H y \quad (6)$$

Where  $I$  is the identity matrix The MMSE based detector [24], output is given by:

$$L(c_p^u, j) = \frac{1}{\sigma_{j,u}^2} \left( \min_{x \in X_p^0} \|x_{j^u} - x\|^2 - \min_{x \in X_p^1} \|x_{j^u} - x\|^2 \right) \quad (7)$$

Where  $J= 1, 2, 3, \dots, N_{tu}$

The resulting soft detector estimates are passed to bit de-interleavers and then to the channel decoders for each user. CRC mechanism is considered at the receiver end for error detection. Feedback channel is anticipated to be acting like free of errors for sending positive ACK and negative ACK to the transmitter in case of any packet success or failure.

#### IV. Proposed Technique

We analyze SHARQ IR protocol in overloaded MIMO system with LDPC encoder using the following multiple retransmission technique: a user is encoded by LDPC mother code at the transmitter end. Initially, a few encoded bits are transmitted over the channel and decoding is attempted at the

receiver end. When packet errors do occur, negative acknowledgment is conveyed to the transmitter through the feedback channel. During the retransmission there is no need to use the entire wireless channel, only additional redundant information bits are transmitted incrementally to help correct the erroneous bits in the erroneous data packet along with the transmission of the new data packet. This means that new data and the failed data packets are now combined over the MIMO channel for the effective utilization of bandwidth.

At the receiver, after JML detection or MMSE detection LDPC will perform the decoding of received packets. For simplicity at the receiver combining side, we keep the same modulation and the same coding scheme for each retransmission.

Using Equation (1), the received signal vector at the  $g^{th}$  (re)transmission can be given by Equation (8) as:

$$y(g) = H(g)x(g) + v(g) \quad (8)$$

Where  $g=1,2,\dots,G$  is the retransmission number

Similarly, the combined received vectors [21] after  $G$  (re)transmissions can be written as:

$$= \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(G) \end{bmatrix} = \begin{bmatrix} H(1) \\ H(2) \\ \vdots \\ H(G) \end{bmatrix} x + \begin{bmatrix} v(1) \\ v(2) \\ \vdots \\ v(G) \end{bmatrix} \quad (9)$$

The procedure of utilizing SHARQ IR in an overloaded MIMO system is also given in Algorithm 1 and is explained as follows. At time  $t=1$ , all users will be transmitting their data. Here, after checking CRC in the first transmission, all those columns in  $H$  matrix representing the correctly decoded users are set to zeros. The received signal using the modified  $H$  matrix is given by.

$$\tilde{y}(1) = \tilde{H}(1)x(1) + v(1) \quad (10)$$

Secondly, the interval parity bits of erroneous users are retransmitted along with a new message. It means that they contain both new and retransmitted information. At the receiver end, both the signal vectors are stacked, as shown by equation (11).

$$= \begin{bmatrix} \tilde{y}(1) \\ y(2) \end{bmatrix} \quad (11)$$

**Algorithm 1: SHARQ IR**

1. **Initialize:**  $g = 1, \mathbb{Y} = 0$  and  $\mathbb{E} = 0$   
 $g =$  no of iterations of retransmission,  $\mathbb{Y} =$  set of users whose data packets are decoded correctly and  $\mathbb{E} =$  set of users whose data packets are decoded with error
2. **Generate** H sub matrix and perform soft-output JML on matrix received Y (1).
3. **Calculation of CRC** for each data packet
4. **Update**  $g, \mathbb{Y}, \mathbb{E}$  and  $\Psi$  where  $\Psi$  is a set of parity bits in kernel block
5. **Repeat** the process till last kernel block, i.e.,  $\Psi_5$  is transmitted
6. Each user belonging to set  $\mathbb{E}$  will retransmit its packet whereas those users belonging to set  $\mathbb{Y}$  will transmit a new data packet in the next time interval
7. **If**  $\mathbb{E} \neq 0$  **then** update  $g = g + 1$  and transmit the first kernel block  $\Psi_1$
8. Received signal matrix Y are then stack with the previous received matrices
9. **Perform** MUD on the resulting matrix to obtain LLR and calculate CRC for each decoded packet
10. **Update**  $\mathbb{Y}$  and  $\mathbb{E}$
11. **If**  $g=G$  and  $\mathbb{Y} \neq 0$  **then** consider that packet as a drop packet
12. All users will transmit new packet

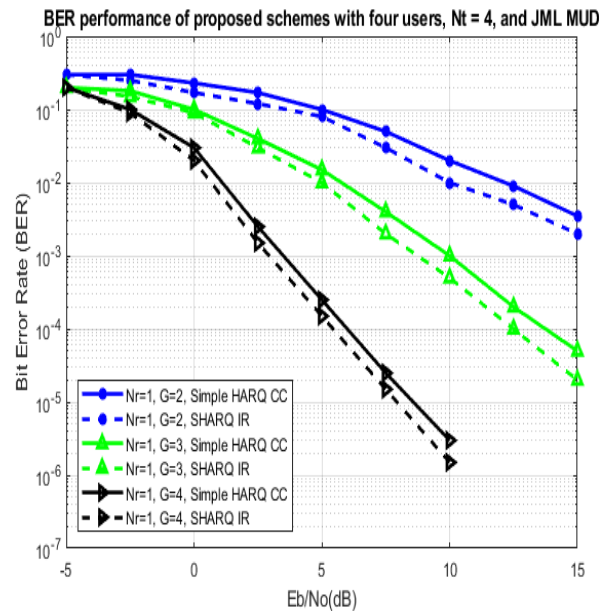
This process continues until the correction of all data bits or the maximum number of G (retransmission) is received. The packet is treated as lost if decoding fails and the maximum G is reached.

**V. SIMULATION RESULTS**

Simulation results are implemented using 4 x 1 MIMO system with 4 users and 1 centralized receiver. Each packet contains 576 bits encoded at a rate of 1/2 LDPC code using 16-QAM modulations. A maximum number of retransmission G is considered to be 4 after which a packet is considered as a dropped packet. The performance parameters for SHARQ IR are evaluated in terms of BER and throughput and matched the performance with that of HARQ Chase Combining (HARQ CC).

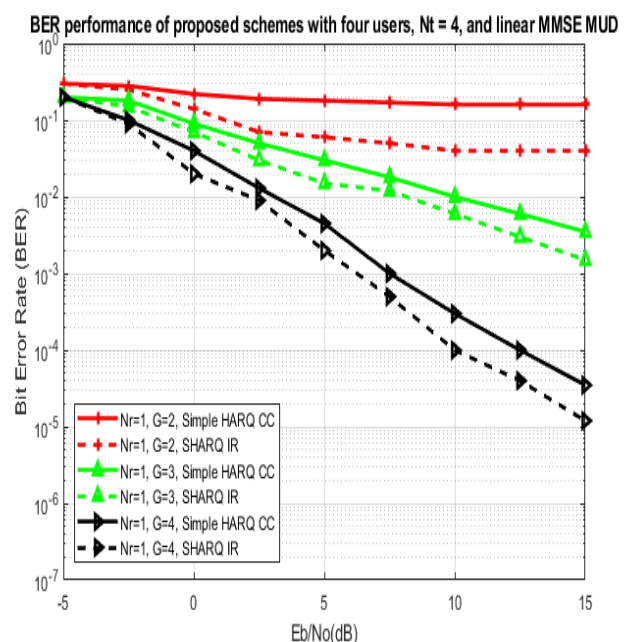
The resulting BER vs SNR (Eb/No) performance for an overloaded MIMO system (i.e., four transmitters and single receiver) using SHARQ IR and JML detection is shown in Figure 2. The result clearly shows that SHARQ IR outperforms HARQ CC when the number of retransmission increases. BER for the SHARQ IR is still better even when we are retransmitting the new bits in the next time slot. For example, at SNR of 10 (Eb/No) for G=2, SHARQ IR BER is 0.01 while that conventional HARQ CC is 0.015. This means that by using SHARQ IR, BER is still better by the factor of 0.005 using JML detection. Further by increasing the

number of retransmission, i.e. G =4, BER at 10 SNR for SHARQ IR is 1.25 while for HARQ CC it is 1.99 a further improvement by a factor of 0.74 is observed. It is important to reiterate that cumulating G means increasing sending the new packets with the erroneous data.



**Figure 2 BER comparison of SHARQ IR and HARQ CC using JML detection**

Similarly, in Figure 3, SHARQ IR is evaluated in terms of BER with HARQ CC and is shown using MMSE detection technique. Here also SHARQ IR performs better than HARQ Chase Combining CC for each transmission and better BER is achieved.



**Figure 3 Figure 3: BER of SHARQ IR and HARQ CC using MMSE detection**

As illustrated at SNR of 10 db for G=2, BER for SHARQ IR is 0.010 while that for HARQ CC BER is 0.015. It is also important to highlight that 4 x 1 overloaded system with G=2 performs poorly as compared to G=3 and 4. This is because the scheme allows transforming the overloaded system to either critically loaded or under loaded MIMO system using a sufficient number of retransmissions. However, by increasing the number of retransmission the throughput gain is affected as well. Similarly, a higher value of G means more number of retransmissions and more cancellation of correctly decoded packets and thereby resulted in an increase in the SINR and permitting to apply MMSE during overloaded conditions.

The vital role of this paper is to improve throughput under the channel condition when it is noisy. It is given by equation (12) as

$$\delta = \frac{\log_2 \omega R (1 - P_{rate})}{N_{avg}} \quad \text{B/s/Hz} \quad (12)$$

$N_{avg}$  is the maximum number of retransmission,  $P_{rate}$  :

is drop packet rate and  $R$  is the code rate. SHARQ IR throughput efficiency is high as compared to HARQ CC because SHARQ IR has this capability to adopt itself to error correction under the varying channel conditions. Figure 4 shows throughput comparison of SHARQ IR with HARQ CC when G is 2 and Nr is also 2.

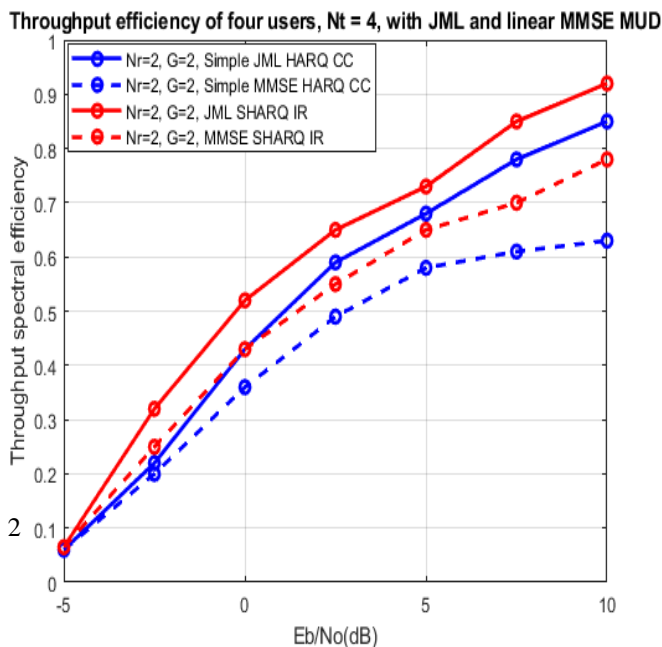


Figure 4 Throughput comparison of SHARQ IR and HARQ CC

As shown in Figure 4, SHARQ IR throughput is enhanced very much as compared to HARQ CC. For example, at 10 Eb/No (dB), the throughput of SHARQ IR is 0.92 using JML detection and is 0.85 for simple HARQ CC. Similarly, for

MMSE detection at SNR of 10 Eb/No (dB), throughput gain of 0.78 is achieved for SHARQ IR as compared to simple HARQ CC which is 0.62 respectively. Throughput efficiency for the same scheme is also implemented and compared using G= 4 and Nr=1 as shown in Figure 5.

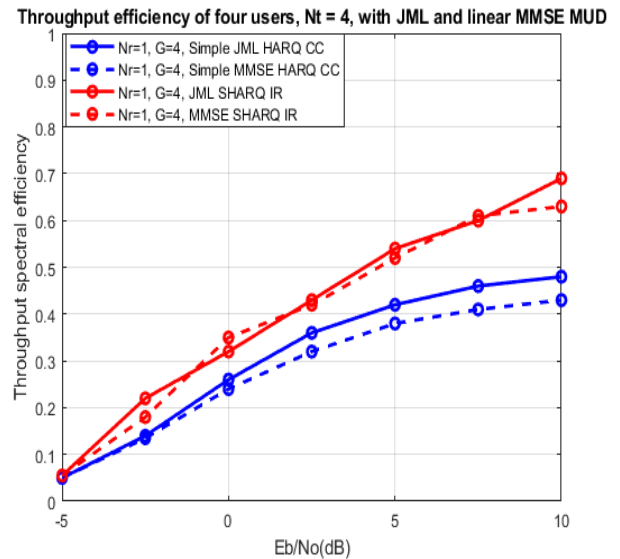


Figure 5 Throughput comparison of SHARQ IR and HARQ CC

As illustrated SHARQ IR throughput for JML detection using G=4 and Nr=1, at SNR of 10 Eb/No (dB) is 0.68 whereas for simple HARQ CC it is 0.48 using JML detection. It means an increase of 20% efficiency is achieved when we increase the number of retransmissions. Moreover, for MMSE at SNR of 10 Eb/No (dB), the throughput of 0.64 is achieved for SHARQ IR as compared to simple HARQ CC, which is 0.44. This means that the overall throughput of the system by deploying SHARQ IR in MIMO system considering an overloaded case is improved by a factor of 20%. This enhanced performance of SHARQ IR is due to the circumstances that contemporary to HARQ CC, SHARQ IR is now sending the parity bits incrementally along with the new data bits during each (re)transmission..

## VII. CONCLUSION

In this paper, SHARQ IR scheme has been proposed in an overloaded MIMO system where an overloaded MIMO system is transformed into a critically or under loaded MIMO system through simple retransmission (using simple linear MUD algorithms). We presented an approach for SHARQ IR in an overloaded MIMO system to conserve the bandwidth by combining both the data bits and parity bits along the wireless channel. The proposed approach is envisaged ultimately to support energy-efficient transmissions for Green IoT Applications. The performance of SHARQ IR scheme is evaluated using BER and throughput while

comparing it with conventional HARQ CC scheme. Better results both in terms of BER and throughput have been achieved with the proposed scheme by adding no complexity to the overall system.

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